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Queuing Model for Healthcare Services in Public Health Facilities (A Case Study of Muhima Hospital)

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Abstract: **The purpose of this project was to analyze time that patients can spend waiting for service in Muhima District hospital. The main objective for this research was to provide necessary information to policy makers aimed to contribute in wellbeing of population by reducing waiting time for service because in excessive cases, long queues can delay appropriate decision for a specific disease that can cause occurrence of death while patient still waiting for service. This project examined, first the waiting time of patients in outpatient department by using queuing model after calculating the mean number of arrivals per hour and the mean number of patients served per hour. Further results from questionnaire from staff were analyzed in order to know their opinions about the waiting time in outpatient department. The results showed that the system utilization factor is greater than one. This means that the queue grew without bound. There were a big number of patients waiting in the queue and they waited for a long time before being seen by a physician. The correlation analysis revealed a significant negative correlation between days and patient arrivals which means that there were many patients on Monday more than Friday. The reasons given by staff interviewed were the big number of patients visiting this department and the shortage of staff. To reduce the waiting time, we suggested increasing the number of physicians and nurses in outpatient department and strengthening the capacity building of health care providers. The hospital should develop a staffing plan and put more effort in the beginning of the week for efficient use of available resources.**

Keywords: **Queuing model, Waiting time, Customer arrival, Customer service, Birth and Death process, Outpatient Department.**

I. INTRODUCTION

1.1 Statement of the Problem:

The service facilities whose customers are patients vary generally in capacity and size, from small outpatient clinics to large, urban hospitals to referral hospitals. Regardless these differences, healthcare processes can be categorized based on how patients arrive, wait for service, obtain service, and then depart.

The healthcare processes also vary in complexity and extent, but they all deal with a set of both medical and non-medical activities and procedures that the patient must experience before getting the desired treatment. The servers in hospital queuing systems are the trained staff and equipment required for specific activities and procedures.

Almost all of us have waited for many hours, many days or many weeks to get an appointment with a medical doctor, and at arrival we are obliged to wait for a long time until being seen. In hospitals, it is not strange to get patients waiting for radiologist for imaging diagnosis and delays for surgery appointment.

Queues are everywhere, particularly in hospitals; lengthy queues are unfavorable for patients because delay in accessing needed services often cause prolonged pain and economic failure when patients are not able to work and potential deterioration of their medical conditions that can augment consequent treatment expenses and poor health outcomes. In excessive cases, long queues can delay appropriate decision for a specific disease that can cause occurrence of death while patient still wait for service.

Therefore, queuing has become a sign of incompetence of public hospitals in the world and Rwanda is not an exception. Decrease of waiting time of patients for healthcare service is one of the challenges facing the majority of hospitals. A few of the factors that is responsible for long waiting lines or delays in providing service are: lack of passion and commitment to work on the part of the hospital staff, overloading of available doctors, doctors attending to patients in more than one clinic etc (Belson, 1988).

These put doctors under stress and tension, hence tends to dispose off a patient without in-depth probing or treatment, which often leads to patient dissatisfaction (Babes, 1991).

This project is based on the perceptive that most of these challenges can be managed by using queuing model to determine the waiting line performance such as: average arrival rate of patients, average service rate of patients, system utilization factor and the probability of a specific number of patients in the system. The resulting performance variables can be used by the policy makers to increase competence, improve the quality of patient care and reduce cost in hospital institutions as well.

1.2 Objectives:

1.2.1 General Objective

The main objective of this project was to apply a queuing model for healthcare services in Muhima District Hospital.

1.2.2 Specific Objectives

This project had the following specific objectives:

1. To determine the mean number of arrivals per hour (λ) in Muhima District hospital.
2. To determine the mean number of patients served per hour (μ) in Muhima District hospital.
3. To compare the mean number of arrivals and the mean number of patients served per hour (λ and μ) in Muhima District hospital.
4. To determine the average time a patient spends waiting in the queue before being seen by a physician in Muhima District hospital.
5. To analyze the waiting line of patients in Muhima District hospital.

1.3 Research Questions:

1. What is the mean number of arrivals per hour (λ)?
2. What is the mean number of patients served per hour (μ)?
3. What is the relationship between the mean number of arrivals and the mean number of patients served per hour (λ and μ)?
4. What is the average time a patient spends waiting in the queue before being seen by a physician?
5. What resources needed to reduce the length of queues in hospitals and increase patients' satisfaction?

1.4 Justification:

This research was conducted in order to fulfill the requirements for the award of the degree of Master of Science in Applied Statistics and it was benefit in different ways:

It will increase the knowledge of the student by relating the theories encountered from lectures to the real world of application. It will also contribute to increase patients' satisfaction in public health facilities. The decision makers in health system will benefit from this research by using results to develop their staffing plan. This project will serve as reference for other researchers in this field by filling the gaps encountered in present research.

1.5 Scope:

This research has been conducted to the patients visiting MUHIMA hospital in Outpatient Department (OPD) for consultation by a physician. A period of 36 days has been covered in which five days of each week from Monday to

Friday were considered because they are the working days of the week, from 08:00 A.M to 12:00 AM and from 01:00 P.M to 05:00 P.M. A questionnaire has been conducted to the nurses and physicians of the Outpatient Department to collect their opinions about causes and proposed solutions of queues.

2. METHODOLOGY

2.1 Birth and Death Process Queuing Models:

A number of important queuing theory models fit the birth-and-death process. A queuing system based on the birth-and–death process is in state En at time t if the number of customers is then n, that is, N(t)=n. A birth is a customer arrival, and a death occurs when a customer leaves the system after completing service. Thus, given the birth rates $\{\lambda_n\}$ and death rate $\{\mu_n\}$, and assuming that

$$S=1 + C1 + C2 + C3 + \ldots < \infty \tag{1}$$

Where

$$C_n = \frac{\lambda_0 \lambda_1 \cdots \lambda_{n-1}}{\mu_1 \mu_2 \cdots \mu_n}, \qquad n= 1, 2, 3, \ldots, \tag{2}$$

We calculate

$$P0 = 1/S \tag{3}$$

and

$$Pn= P[N=n] = CnP0, \qquad n= 1, 2, 3, \ldots \tag{4}$$

From the probabilities calculated by (4) we can generate measures of queuing system performance (Lajos, 1962).

2.2 M/M/1 Queuing System:

Figure.1: M/M/1 Queuing System.

This model assumes a random (Poisson) arrival pattern and a random (exponential) service time distribution. The arrival rate does not depend upon the number of customers in the system and the probability of an arrival in a time interval of length h>0 is given by

$$\begin{aligned} e^{-\lambda h}(\lambda h) &= \lambda h\left(1-\lambda h+\frac{(\lambda h)^2}{2!}-\ldots\right) \\ &= \lambda h-(\lambda h)^2-\frac{(\lambda h)^3}{2!}-\ldots+(-1)^{n+1}\frac{(\lambda h)^n}{(n-1)!}+\ldots \\ &= \lambda h + o(h) \end{aligned} \tag{5}$$

Thus, we have

$$\lambda_n = \lambda \qquad n= 0, 1, 2, \ldots \tag{6}$$

By hypothesis, the service time distribution is given by

$$W_s(t) = P[s \le t] = 1-e^{-\mu t}, \qquad t \ge 0. \tag{7}$$

Then, when a customer is receiving service, the probability of a service completion (death) is a short time interval, h, is given by

$$1-e^{-\mu h} = 1-\left(1-\mu h+\frac{(\mu h)^2}{2!}-\ldots\right) = \mu h + o(h). \tag{8}$$

(Here we have used the memoryless property of the exponential distribution in neglecting the service already completed.) (Little, 1961).

Thus,

$$\mu_n = \mu, \qquad n=1, 2, 3, \ldots. \tag{9}$$

Thus the state-transition diagram for the M/M/1 queuing system is given by Figure 2 and therefore, by (1), since $\lambda / \mu = \rho,$ and each C_n is equal to $\rho^n,$

$$S = 1 + \rho + \rho^2 + \ldots + \rho^n + \ldots = 1/(1-\rho). \tag{10}$$

Hence,

$$p_n = P[N=n] = (1-\rho)\rho^n, \qquad n=0, 1, 2, \ldots \tag{11}$$

Figure.2: State-transition diagram of the M/M/1 *queuing system.*

But (11) is the pmf for a geometric random variable, that is, N has a geometric distribution with $p = 1-\rho$ and $q = \rho.$ Hence,

$$L = E[N] = q/p = \rho/(1-\rho), \tag{12}$$

and

$$\sigma_N^2 = \rho \big/ (1-\rho)^2. \tag{13}$$

By Little's formula,

$$W = E[q] = W - E[s] = \rho E[s]/(1-\rho), \tag{14}$$

since $\rho = \lambda E[s].$

Now,

$$W_q = E[q] = W - E[s] = \rho E[s]/(1-\rho). \tag{15}$$

Applying Little's formula, again, gives,

$$L_q = E[N_q] = \lambda W_q = \rho^2 \big/ (1-\rho). \tag{16}$$

By (11) we calculate

P[server is busy]=1-P[N=0]=1-(1-ρ)=ρ.

By the law of large numbers this probability can be interpreted as the fraction of time that the sever is busy; it is appropriate to call ρ the "server utilization.

We now have the four parameters most commonly used to measure the performance of a queuing system, W, Wq, L and Lq as well as the pmf, p_n, of the number in the system.(Allen, 1978).

2.3 The M/M/1/K Queuing System:

The M/M/1/K system is a more accurate model of this type of system in which a limit of K customers is allowed in the system. When the system contains K customers, arriving customers are tuned away. Figure 3 is the state-transition diagram for this model. Thus, a birth-and-death process, the coefficients are (Parzen, 1962)

$$\lambda_n = \begin{cases} \lambda & \text{for } n=0, 1, 2, \ldots, k-1 \\ 0 & \text{for } n \geq K \end{cases} \tag{17}$$

and

$$\mu_n = \begin{cases} \mu & \text{for } n=0, 1, 2, \ldots, K \\ 0 & \text{for } n>K \end{cases} \tag{18}$$

This gives the steady state probabilities

$$p_n = \left(\frac{\lambda}{\mu}\right)^n p_0 = u^n p_0 \qquad \text{for } n=0, 1, 2, \ldots, K, \tag{19}$$

Where

$$u = \lambda E[s] = \lambda / \mu.$$

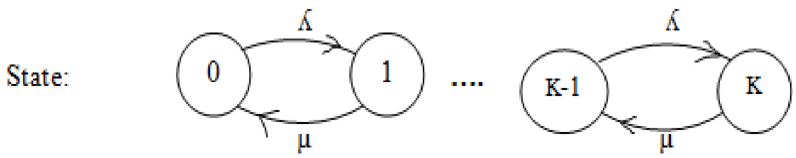

Figure.3: State-Transition diagram for the M/M/1/K queuing system.

Since

$$1 = p_0 + p_1 + \ldots + p_K = p_0 \sum_{n=0}^{K} u^n = \left(\frac{1-u^{K+1}}{1-u}\right) p_0,$$

if $\lambda \neq \mu,$ we have,

$$p_0 = (1-u)/(1-u^{K+1}) \tag{20}$$

Since there are never more than K customers in the system, the system reaches a steady state for all values of λ and μ. That is, we need not assume that $\lambda < \mu$ for the system to achieve a steady state. If $\lambda = \mu$ then $\rho = 1$ and

$p_0 = 1/(K+1) = p_n$ for n=1, 2, …, K

Thus the steady state probabilities are

$$p_n = \begin{cases} \dfrac{(1-u)u^n}{1-u^{K+1}} & \text{for } \lambda \neq \mu, \quad n=0, 1, \ldots, K \\ \dfrac{1}{K+1} & \text{for } \lambda = \mu, \quad n=0, 1, \ldots, K \end{cases} \tag{21}$$

It should be noted that, if $\lambda < \mu,$ as $K \to \infty,$ each p_n in (20) approaches the value in (11), as it should (Gross, 1974).

If $\lambda \neq \mu$ then

$$L = E[N] = \sum_{n=1}^{K} np_n = \sum_{n=1}^{K}\left(\frac{1-u}{1-u^{K+1}}\right)(nu^n) = \left(\frac{1-u}{1-u^{K+1}}\right) u \sum_{n=1}^{K} nu^{n-1}$$

$$= \left(\frac{1-u}{1-u^{K+1}}\right) u \sum_{n=1}^{K} \frac{du^n}{du} = \left(\frac{1-u}{1-u^{K+1}}\right) u \frac{d}{du} \sum_{n=0}^{K} u^n$$

Continuing calculations in the same way we get

$$L = \frac{u}{1-u} - \frac{(K+1)u^{K+1}}{1-u^{K+1}}. \tag{22}$$

Thus if $\lambda < \mu$, the expected number in the system, L, is always less than for the unlimited queue length case (where L is $u/(1-u)$).

If $\lambda = \mu$ then $u = 1$ and

$$L = \sum_{n=1}^{K} np_n = \frac{1}{K+1}(1+2+...+K) = \frac{K(K+1)}{2(K+1)} = \frac{K}{2}. \tag{23}$$

Thus (21) and (4) can be summarized by

$$L = \begin{cases} \dfrac{u}{1-u} - \dfrac{(K+1)u^{K+1}}{1-u^{K+1}} & \text{if } \lambda \neq \mu \\ \dfrac{K}{2} & \text{if } \lambda = \mu \end{cases} \tag{24}$$

In either case,

$$L_q = L - (1 - p_0) \tag{25}$$

because

$$E[N_S] = P[N=0]E[N_S | N=0] + P[N>0]E[N_S | N>0]$$

$$= p_0 * 0 + (1-p_0) * 1 = 1 - p_0.$$

All the traffic reaching the system does not enter the system because customers are not allowed admission when there are K customers in the system, that is, with probability p_K. Thus, if λ_a is the average rate of customers into the system, (Karlin, 1969)

$$\lambda_a = \lambda(1 - p_k). \tag{26}$$

We can then apply Little's formula to obtain

$$W = E[w] = L/\lambda_a, \tag{27}$$

and

$$W = E[q] = L_q/\lambda_a, \tag{28}$$

The true server utilization, ρ, which is the probability that the server is busy, is given by

$$\rho = \lambda_a E[s] = \lambda(1-p_K)E[s] = (1-p_K)u. \tag{29}$$

2.4 The M/M/c Queuing System:

For this model we assume random (exponential) interarrival and service times with C identical servers (Kleinrock, 1975). This system can be modeled as a birth-and-death process with the coefficients

$$\lambda_n = \lambda, \qquad \text{n=0, 1, 2, ...,} \tag{30}$$

and

$$\mu_n = \begin{cases} n\mu, & \text{n=1,2,3,...,c} \\ c\mu, & \text{n=c, c+1, ...} \end{cases} \tag{31}$$

The state-transition diagram is shown in Figure. (5.2.4).

Thus, by (2), with $u=\lambda/\mu$ and $\rho=u/c$,

$$S=\frac{1}{p_0}=1+u+\frac{u^2}{2!}+\ldots+\frac{u^{c-1}}{(c-1)!}+\frac{u^c}{c!}\left(1+\frac{u}{c}+\left(\frac{u}{c}\right)^2+\ldots\right)$$

$$=\sum_{n=0}^{c-1}\frac{u^n}{n!}+\frac{u^c}{c!}\sum_{n=0}^{\infty}\rho^n=\sum_{n=0}^{c-1}\frac{u^n}{n!}+\frac{u^c}{c!(1-\rho)}. \qquad (32)$$

Hence, $$p_0=\left[\sum_{n=0}^{c-1}\frac{u^n}{n!}+\frac{u^c}{c!(1-\rho)}\right]^{-1} \qquad (33)$$

State:

Figure.4: State-transition diagram for $M/M/c$ queuing system.

and

$$p_n=\begin{cases}\dfrac{u^n}{n!}p_0 & \text{if n=0, 1, 2, ...,c.}\\[2mm] \dfrac{u^n}{c!c^{n-c}}p_0 & \text{if } n\ge c\end{cases} \qquad (34)$$

We will now derive the primary measures of system performance, Lq, Wq, W and L.

By definition, (Köollerström, 1974)

$$L_q=E\left[N_q\right]=\sum_{n=c}^{\infty}(n-c)p_n=\sum_{k=0}^{\infty}kp_{c+k}=\sum_{k=0}^{\infty}k\frac{u^c}{c!}\rho^k p_0=p_0\frac{u^c}{c!}\sum_{k=0}^{\infty}k\rho^k$$

$$=p_0\frac{u^c}{c!}\left\{0+1\rho+2\rho^2+3\rho^3+\ldots\right\}=p_0\frac{u^c}{c!}\rho\frac{d}{d\rho}\left\{1+\rho+\rho^2+\ldots\right\}$$

$$=p_0\frac{u^c}{c!}\rho\frac{d}{d\rho}\left(\frac{1}{1-\rho}\right)=\frac{p_0u^c\rho}{c!(1-\rho)^2}. \qquad (35)$$

Equation (35) can also be written as

$$L_q=\frac{\rho^{c+1}}{(c-1)!(c-\rho)^2}*p_0 \qquad (36)$$

Having computed Lq by the formula (35), we can calculate

$$W_q=L_q/\lambda \qquad (37)$$

$$W=W_q+E[s]=W_q+(1/\mu), \qquad (38)$$

and

$$L=\lambda W \qquad (39)$$

From Equation (37), (39) can be expressed as

$$L=L_q+\lambda/\mu \qquad (40)$$

While we have the pmf of the number in the system, N, and the expected values of the primary random variables, it is useful to have the distribution functions of w and q. We will drive $W_q(.)$ and state the formula for $W(.)$.

First, we note that

$$W_q(0)=P[q=0]=P[N\le c-1]=\sum_{n=0}^{c-1} p_n = p_0 \sum_{n=0}^{c-1}\frac{u^n}{n!}. \tag{41}$$

But, by (33)

$$p_0\left(\sum_{n=0}^{c-1}\frac{u^n}{n!}\right)+\frac{p_0 u^c}{c!(1-\rho)}=1$$

Or

$$p_0\left(\sum_{n=0}^{c-1}\frac{u^n}{n!}\right)=1-\frac{p_0 u^c}{c!(1-\rho)}.$$

Therefore, we have

$$W_q(0)=1-\frac{p_0 u^c}{c!(1-\rho)}=1-\frac{p_c}{1-\rho}. \tag{42}$$

Now, suppose $N=n\ge c$ when a customer arrives. All c servers are busy so the time between service completions has an exponential distribution with average value $1/c\mu$.

There are c customers receiving service and n-c customers waiting in the queue. Therefore, the new arrival must wait for $n-c+1$ service completions before receiving service. (If n=c, so no customer is waiting, the new arrival must wait for one service completion. If n=c+1, two service completions are required, etc.)

Hence, the waiting time in queue is the sum of n-c+1 independent exponential random variables each with mean $1/c\mu$; that is it is gamma with parameters $n-c+1$ and $c\mu$. Hence, if t>0, we can write, by (18), since

$\Gamma(n-c+1)=(n-c)!,$ that (Hillier, 1971)

$$\begin{aligned}
W_q(t) &= W_q(0)+\sum_{N=c}^{\infty} p_n P\left[q\le t \middle| N=n\right]\\
&= W_q(0)+\sum_{N=c}^{\infty} p_0\frac{u^n}{c!c^{n-c}}\int_0^t \frac{c\mu(c\mu x)^{n-c}}{(n-c)!}e^{-c\mu x}dx\\
&= W_q(0)+p_0\frac{u^c}{(c-1)!}\int_0^t \mu e^{-c\mu x}\left(\sum_{n=c}^{\infty}\frac{(\mu u x)^{n-c}}{(n-c)!}\right)dx\\
&= W_q(0)+\frac{p_0 u^c}{(c-1)!}\int_0^t \mu e^{-c\mu x}e^{-\mu u x}dx\\
&= W_q(0)+\frac{p_0 u^c}{(c-1)!}\int_0^t \mu e^{-\mu x(c-u)}dx\\
&= W_q(0)+\frac{p_0 u^c}{(c-u)(c-1)!}\left(1-e^{-\mu t(c-u)}\right)\\
&= 1-\frac{p_0 u^c}{(c-u)(c-1)!}+\frac{p_0 u^c}{(c-u)(c-1)!}\left(1-e^{-\mu t(c-u)}\right)\\
&= 1-\frac{p_0 u^c}{(1-\rho)c!}e^{-\mu t(c-u)}\\
&= 1-\frac{p_c}{(1-\rho)}e^{-\mu t(c-u)}=1-\frac{p_c}{(1-\rho)}e^{-c\mu t(c-\rho)}.
\end{aligned} \tag{43}$$

The quantity $p_c/(1-\rho)$ is an interesting quantity; in fact it is the probability that an arriving customer must wait; it is known as Erlang's C formula or Erlang's delay formula and written

$$C(c,u)=\frac{u^c}{c!(1-\rho)}p_0=\frac{u^c/c!}{(1-\rho)\left[\left(\sum_{n=0}^{c-1}\frac{u^n}{n!}\right)+\frac{u^c}{c!(1-\rho)}\right]}, \tag{44}$$

Where, of course, $u=\lambda/\mu$ and $\rho=u/c$.

To see that Erlang's C formula, (44), does give the probability that an arriving customer must wait we note that this probability is

$$\sum_{n=c}^{\infty}p_n=1-\sum_{n=0}^{c-1}p_n=1-W_q(0)=\frac{p_c}{1-\rho},$$ by (42).

Hence (41) can be written as

$$W_q(t)=1-P[\text{arriving customer must queue}]e^{-c\mu(1-\rho)}$$

$$=1-C(c,u)e^{-c\mu t(1-\rho)},\qquad t\geq 0 \tag{45}$$

Formula (43) can be used to calculate the r^{th} percentile value of q, π_q (r).

The distribution function W(.) for the waiting time is given by

$$W(t)=1+\frac{(u-c+W_q(0))}{c-1-u}e^{-\mu t}+\frac{C(c,u)}{c-1-u}e^{-c\mu t(1-\rho)},\qquad \text{if } u\neq c-1 \tag{46}$$

and by

$$W(t)=1-[1+C(c,u)\mu t]e^{-\mu t},\qquad \text{if } u=c-1 \tag{47}$$

2.5 The M/M/c/c Queuing System:

This system is sometimes called the M/M/C loss system because customers who arrive when all servers are busy are not allowed to wait for service and are lost. The state-transition diagram is given in Figure 4.

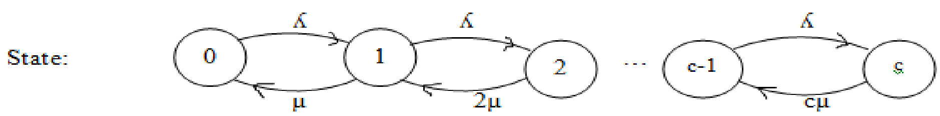

Figure.5: State-transition diagram for $M/M/c/c$ queuing system.

From the diagram we see that

$$C_n=u^n/n!,\qquad n=1,\ 2,\ \ldots,\ c, \tag{48}$$

Where, as usual, $u=\lambda E[s]=\lambda/\mu$, and

$$S=1/p_0=1+u+u^2/2!+\ldots+u^c/c!. \tag{49}$$

Thus

$$p_n=\frac{u^n/n!}{1+u+u^2/2!+\ldots+u^c/c!}=B(c,u).\qquad n=0,\ 1,\ 2,\ \ldots,\ c. \tag{50}$$

The distribution given by (50) is called "truncated Poison distribution," for obvious reasons (Kingman, 1962). In particular, the probability that all servers are busy, so that an arriving customer is lost, is

$$p_n = \frac{u^n/n!}{1+u+u^2/2!+\ldots+u^c/c!} = B(c,u). \tag{51}$$

$B(c,u)$ is called "Erlang's B formula" or "Erlang's loss formula" in honor of It's discoverer, A.K.Erlang. Just as with M/M/1/K model the actual average arrival rate into the system, λ_a, is less than λ because some arrivals are tuned away. We must have

$$\lambda_a = \lambda(1-B(c,u)). \tag{52}$$

Since no customers are allowed to wait, W_q and L_q are zero. However,

$$L = E[N] = \sum_{n=0}^{c} np_n = p_0 \sum_{n=1}^{c} n\frac{u^n}{n!} = u\,p_0 \sum_{n=0}^{c-1} \frac{u^n}{n!} = u(1-B(c,u)). \tag{53}$$

By Little's formula,

$$W = E[w] = L/\lambda_a = E[s]. \tag{54}$$

Of course (54) is obvious because there is no waiting. Thus w has the same distribution as s (Kleinrock, 1976).

This means that

$$W(t) = P[w \le t] = 1-e^{-\mu t} = 1-e^{-t/E(s)}. \tag{55}$$

2.6 M/M/∞ Queuing System:

No real life queuing system can have an infinite number of servers; what is meant, here, is that a server is immediately provided for each arriving customer. The state-transition diagram for this model is shown in Figure 5. We can read off from the figure that (Allen, 1975)

$$C_n = u^n/n!, \qquad n = 1,2,3,\ldots$$

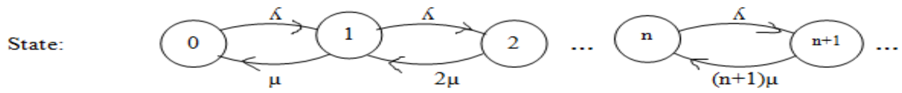

Figure.6: State-transition diagram for $M/M/\infty$ queuing system

so that

$$S = 1/p_0 = \sum_{n=0}^{\infty} u^n/n! = e^u.$$

Hence,

$$p_n = e^{-u}(u^n/n!), n = 0,1,2,\ldots \tag{56}$$

that is, N has a Poison distribution! The fact that p_n has a Poisson distribution tells us that $L = E[N] = u$ is the average number of busy servers, with $Var[N] = u$. The $M/M/\infty$ queuing model can be used to estimate the number of lines in use in a large communication network or as a gross estimate of values in an $M/M/c$ or $M/M/c/c$ queuing system.

2.7 Specification of the model:

In this study, $M/M/c$: FCFS/∞/∞ has been used, where;

M=Markovian (or poisson) arrivals and exponential service time.

$c =$ Multi-server; where in our case c is equal to three physicians working in outpatient department.

FCFS = First come, first served;

∞ = Infinite system limit;

∞ = Infinite source limit.

For the purpose of modeling, the arrivals (n) are the outpatients. As each reaches the hospital, he/she books for service. If service is rendered immediately he/she leaves the hospital or otherwise joins the queue. The doctors are the servers (c).

The arrival rate, service time and number of servers were the data used for the study that have been collected using observation method. The data collection covered a period of 36 days in which five days of each week from Monday to Friday were considered because they are the working days of the week.

2.8 Procedures of System Parameters Estimation:

The system performance parameters used in this study are defined as follows:

λ: Arrival rate of patients at outpatient department per hour;

μ: Service rate (Length of stay) of patients at outpatient department per hour;

C: Number of doctors (servers) working in outpatient department for consultation.

In this model, there are three parallel physicians

ρ : Outpatient system utilization factor = $\lambda/C\mu$,

Lq: Average number of patients at outpatient department in the queue.

$$L_q = \frac{\rho^{c+1}}{(c-1)!(c-\rho)^2} * p_0 \tag{57}$$

L: Average number of outpatients in the system = $L_q + \lambda/\mu$

Wq: Waiting time of outpatients in the queue = Lq/ λ

W: Waiting time of outpatients in the system = L/ λ

Pn = probability of n outpatients existing in the system.

$$P_n = \begin{cases} \dfrac{(\frac{\lambda}{\mu})^n}{n!} * P_0, & if \;\; 0 < n < C \\ \dfrac{\lambda^n}{c!\,\mu^n c^{n-c}} * P_0, & if \;\; c \le n \end{cases} \tag{58}$$

Po = Possibility of 0 outpatients existing in the system.

$$p_0 = \left[\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!} \left(\frac{1}{1-\frac{\rho}{c}} \right) \right]^{-1} \tag{59}$$

2.9 Population of Inquiry:

For our case, the population of inquiry concerned the patients who are coming for consultation in Outpatient department in Muhima hospital.

2.10 Sampling Frame:

For our case, the sampling frame concerned the patients who were coming for consultation in Outpatient department every day in Muhima hospital.

However, given that the patients arrive randomly and we are most interested by the period between two successive patients, we have taken our sampling frame as number of hours in a year to collect arrival and service data.

2.11 Sample and Sampling Technique:

Because of the inability to achieve individually all units of our statistical universe, we have carried out the sampling technique. According to De Landsheere .G. (1982,p. 382), sampling is the fact of choosing a limited number of individuals, objects, events which the observation allows to draw conclusions applicable to the whole population from which the choice has been made. To determine the sample of our study, we have used the following formula:

$$n = \frac{z^2 p(1-p)}{E^2} \tag{60}$$

Where: Z = Z-score (=1.96 for 95% confidence level)

p = Healthcare service utilization rate in Muhima hospital which corresponds to 86%

E = Margin of Error (Confidence Interval); in our case we have decided to use 0.04

The table below shows different values used in calculation and the corresponding sample size n.

Table 1: Sample size calculations

Z	P	1-p	E	N
1.96	0.86	0.14	0.04	289 hours

$$n = \frac{(1.96)^2 0.86 * (1-0.86)}{(0.04)^2} = 289 hours \tag{61}$$

Here we obtain a sample size of 289 hours that correspond to 36 days if we consider 8 working hours per day. We have also selected a total number of 16 staff to interview on the prepared questionnaire without sampling because this number was small and it was not necessary to take a sample from them; we have interviewed the total staff working in Outpatient department.

2.12 Instruments:

In our work, different documents have been used such as books, reports and electronic sources. All these documents helped us to make the conceptual and theoretical framework of our work as well as to analyze the data and interpret the results. Also, we have use a register to record discrete time for patient arrival and service. For collection of staff opinions, a questionnaire has been used.

2.13 Data Collection Procedure:

In this project the observation technique has been used where we registered the time when every patient enters in outpatient department and a time when he/she comes out from outpatient department. This helped to draw a table used in estimating the average number of patients entered in the system and average number of patients served in one hour. From this we have estimated the remaining performance parameters of the system. These data have been collected for a period of 36 days from Monday to Friday, from 08:00 A.M to 12:00 and from 01:00 P.M to 05:00 P.M. A questionnaire has been used to collect staff opinions about causes and proposed solutions of queues.

2.14 Data Processing and Analysis:

For analysis of our data and interpretation of the results, different computer tools have been used especially Microsoft Excel and SPSS. The data collected using observation technique has been entered in Excel spread sheet for cleaning and convert the recorded time in interval time and then imported in SPSS for analysis where descriptive statistics and significance test have been carried out as well as estimation of different performance parameters describing the behavior of the system.

The data from questionnaire has been directly entered in a designed SPSS sheet for cleaning and analysis. The figures and tables were interpreted in scope predefined objectives in order to make data meaningful and come out with conclusions and recommendations.

3. RESEARCH FINDINGS AND DISCUSSION

3.1 The Mean Number of Arrivals per Hour (λ) and mean number of patients served per hour (μ):

The mean number of arrivals (λ) and patients served per hour (μ) have been calculated from the data collected during 36 days of field visit in Muhima hospital in outpatient department.

Time duration was recorded for each patient arriving for consultation in outpatient department, then the interval time period separating a patient arrival and the next one was calculated. We also calculated the time spent by each patient in physician's office. Finally the average intervals time were calculated. After these calculations we found that λ =14.4978 patients arrived per hour, $\mu = 2.3220$ patients served per hour and the system utilization factor was

$$\rho = \frac{\lambda}{c\mu} = \frac{14.4978}{3*2.3220} = 2.0813 \tag{62}$$

Here we observe that traffic intensity is greater than one then the queue will grow without bound.

4.5 Probability that there is no Outpatient Existing in the System.

$$P_0 = \left[\sum_{n=0}^{c-1} \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!} + \frac{\left(\frac{\lambda}{\mu}\right)^c}{c!\left(1-\frac{\lambda}{c\mu}\right)} \right]^{-1} = 0.09849 \tag{63}$$

Pn = probability of n outpatients existing in the system.

$$P_n = \begin{cases} \dfrac{(\frac{\lambda}{\mu})^n}{n!} * P_0, & if \quad 0 < n < C \\ \dfrac{\lambda^n}{c!\,\mu^n c^{n-c}} * P_0, & if \quad c \le n \end{cases} \tag{64}$$

After calculations we get the following results

Table 2: Probability of n outpatients existing in the system

n	Pn	n	Pn	n	Pn	n	Pn
0	0.098488699	3	0.14798437	6	0.04941204	9	0.0164987
1	0.204980855	4	0.102664783	7	0.034279811	10	0.011446043
2	0.213309503	5	0.071224127	8	0.023781764		

3.2 Average Number of Patients Waiting in the Queue before Being Seen by a Physician (Lq):

This is a number of patients that are expected to be on a queue waiting for consultation by a physician. The value has been found by using the following formula:

$$L_q = \frac{\rho^{c+1}}{(c-1)!(c-\rho)^2} * P_0 \tag{65}$$

After calculations we found **Lq= 7.1129,** this means that at outpatient department of Muhima hospital we can expect to find **7 patients** waiting on the door for a physician.

3.3 Average Number of Patients Waiting in the System (L):

The average number of patients waiting in the system represents the number of patients waiting in the queue before being seen by a physician plus the system utilization factor of the system.

$$L = L_q + \lambda/\mu = 7.1129 + 6.2438 = 13.3566 \quad (66)$$

This means that we can expect 13 patients in the outpatient department including those who are waiting on the queue and those who are being in consultation rooms with physicians.

3.4 Average Time a Patient Wait in the Queue before Being Seen by a Physician (Wq):

When a patient arrives at outpatient department can wait for a certain period if all doctors are busy. In this case, the waiting time can vary from a system to another and in our case we found that the waiting time in the queue by using the following formula:

$$W_q = L_q/\lambda = 7.1129/14.4978 = 0.4906 \text{ hours} \approx 29 \min \quad (67)$$

Here we can see that a patient can wait in the queue around 29 minutes before being seen by a physician.

3.5 Average Time a Patient Spends Waiting in the System (W):

This corresponds to the time a patient can spend in the outpatient department since arrival at physician's room up the time he/she come out from the consultation room; this covers the time a patient spends in the queue before being seen by a doctor and the time a patient spends in consultation room with a doctor. The following formula has been used to find the corresponding value:

$$W = L/\lambda = 13.3566/14.4978 = 0.9213 hours \approx 55.2774 \min \quad (68)$$

Here we can say that the patient can spend 55 minutes in the outpatient department including the time of consultation by a physician.

3.6 Correlation Analysis of Waiting Line of Patients:

The correlation analysis in this context will measure the association between number of patients' arrival and days of the week. The Figure 5 gives an overview on average of variation of arrivals and customers served from Monday to Friday.

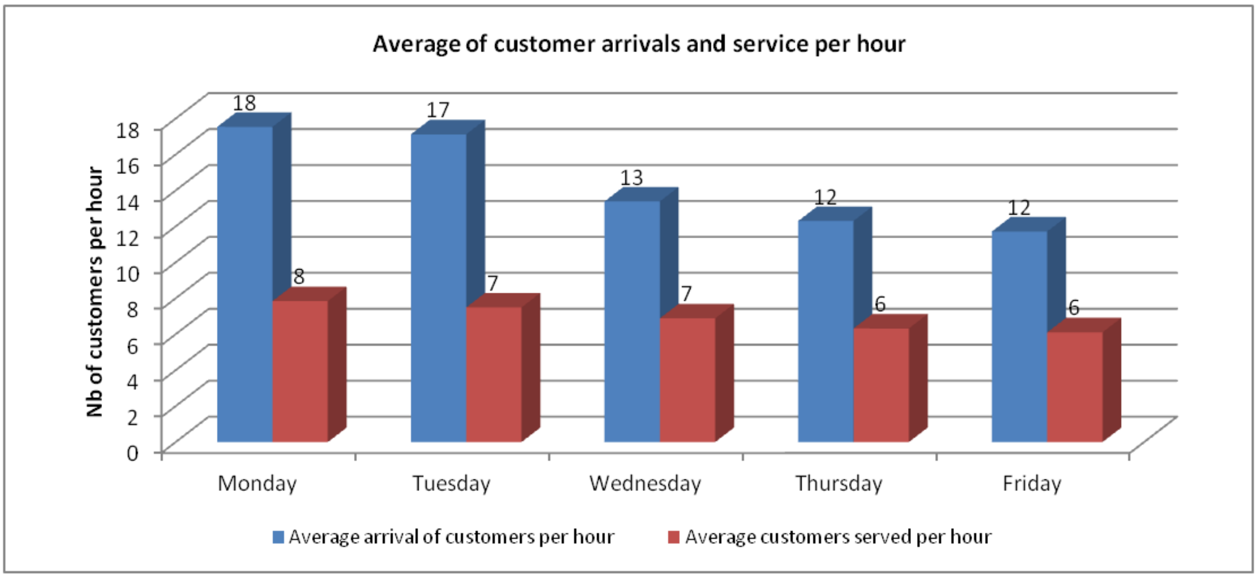

Figure 7: Average of Customer arrivals and service per hour

From Figure 5 we can see that the number of arrivals per hour decline from Monday to Friday 18 patients to 12 patients respectively, while the average number of customers served per hour stay almost stable over the whole week. However we can't conclude on the association based only on this figure, we need to go far and measure this association.

3.6.1 Correlation between Days and Arrivals

The correlation between arrivals over the working days of week has been performed using SPSS software under null hypothesis $\mathbf{H_0}$: *There is no correlation between days and customer arrivals*, and the results displayed as follow:

Table 3: Pearson correlation coefficient of Arrivals and working days of the week

		Days	Arrivals
Days	Pearson Correlation	1	-,933*
	Sig. (2-tailed)		,021
	N	5	5
Arrivals	Pearson Correlation	-,933*	1
	Sig. (2-tailed)	,021	
	N	5	5

*. Correlation is significant at the 0.05 level (2-tailed).

From the Table 5 we find that the Pearson correlation coefficient (r) is -0.933 with small p-value (0.02), therefore we can reject H0; this means that there is a significant negative correlation between days and patient arrivals. In other words there many patients on Monday more than Friday.

A. 4.11 Regression Analysis of Patients' Arrivals over the Days of the Week

This regression has been analyzed using two variables X and Y where X represents working days of the week as independent variable and Y stands for number of arrivals of patients as dependent variable.

$$Y = \alpha + \beta X \tag{69}$$

The significance of coefficients has been tested at 95% confidence interval under the following null hypothesis:

H_0: $\alpha=0$

H_0: $\beta=0$

The following results have been found:

Table 4: Significance test of the regression model

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	19,500	1,256		15,530	,001
	Days	-1,700	,379	-,933	-4,490	,021

a. Dependent Variable: Arrivals

Using the above table we can write the regression model as follow:

$$Y = 19.500 - 1.700X \tag{70}$$

All constants α and β are statistically significant because p-value are small, 0.001 and 0.021 respectively for α and β; this means that the model is significant. This model can be used for prediction of number of patients expected to come for outpatient consultation from Monday to Friday and from this can be deducted an efficient use of resources.

3.7 Analysis of the Questionnaire:

The questionnaire was addressed to the nurses and physicians in outpatient department.

The purpose of this questionnaire was to collect additional information about the causes of the queue in their department and proposed solutions to reduce the queue.

In order to get complementary information guiding to the solutions to reduce the queue, we have asked the nurses and physicians different questions including the following: *"How long do you think a patient can spend waiting before being seen by a physician?"*

Results shown that 37.5% of respondents said that a patient can spend more than one hour waiting before being seen by a physician, 37.5% said that patients can spend between 5 and 10 minutes, while 12.5% said that patients can spend between 10 and 30 minutes and only 12.5% said that patients can spend less than 5 minutes. We need also to know if the estimated time a patient can spend waiting before being seen by a physician was convenient by using the question: *"Do you think that this time is convenient and comfortable for the patients".* From Table 8, the answers have been given:

Table 5: Convenience and comfort of patients on waiting Time

	Frequency	Percent	Valid Percent	Cumulative Percent
Valid No	16	100,0	100,0	100,0

Here we can see that all respondents said "No", the waiting time was not convenient and not comfortable for the patients. Then from this, we were curious to know what they think can be causes of this uncomfortable waiting time by asking: *"what are causes of the long waiting time in this department?"* and the answers were as follows:

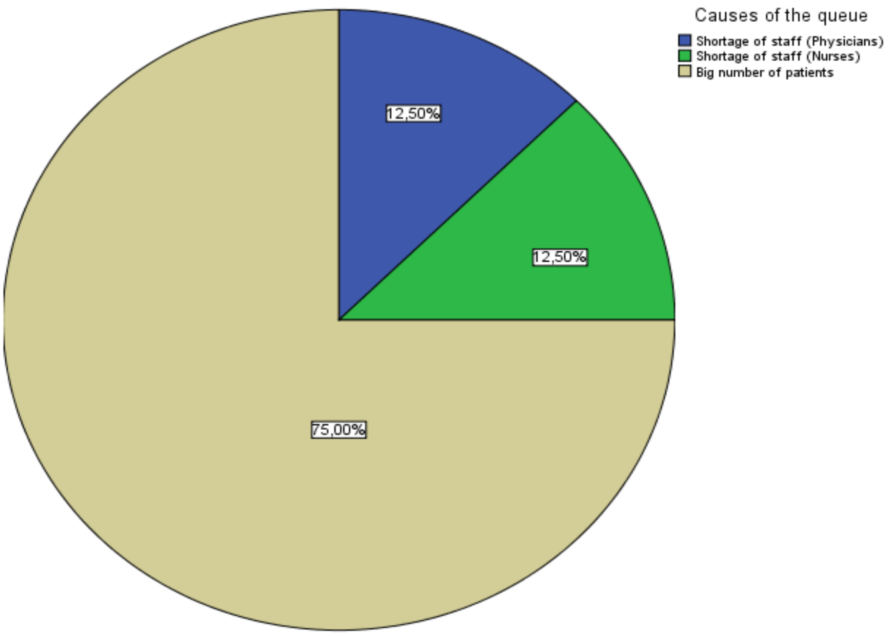

Figure 8: Causes of the long waiting time in OPD

The interviewed nurses and physicians gave three reasons of the long waiting time in outpatient department where 75% said that the cause is a big number of patients visiting this department while 25% attribute this long waiting time to the shortage of staff. However, this is not enough if we don't ask the proposed solutions on this issue.

The question was: *"What Solutions do you think can be proposed to reduce the waiting time of patients in this department"*. The interviewed staff proposed the solutions as follow:

The 37.5% of respondents proposed to increase number of physicians, 50% proposed to increase number of nurses and 12.5% of respondents proposed to strengthen the capacity building of health care providers.

4. CONCLUSION AND RECOMMENDATIONS

The main objective of this project was to apply a queuing model for healthcare services in Muhima District Hospital.

The findings show that the system utilization factor is greater than one this explains that the queue will grow without bound. There were a big number of patients waiting in the queue and they waited for a long time before being seen by a physician. The correlation analysis has revealed a significant negative correlation between days and patient arrivals which means that there are many patients on Monday more than Friday.

The reasons given by 75% of staff interviewed were the big number of patients visiting this department while 25% attribute this long waiting time to the shortage of staff.

To reduce the waiting time, 37.5% of respondents proposed to increase number of physicians, 50% proposed to increase number of nurses and 12.5% of respondents proposed to strengthen the capacity building of health care providers. The hospital should develop a staffing plan and put more effort in the beginning of the week for efficient use of available resources.

REFERENCES

[1] Allen, A.O. (1978) Probability, Statistics and Queuing Theory: With Computer Applications, Academic Press, Florida.

[2] Barbeau, Michel; Kranakis, Evangelos (2007). Principles of Ad-hoc Networking. John Wiley & Sons Ltd. New Jersey.

[3] Carl-Erik. (2003). Model assisted survey sampling. Springer. pp. New Mexico.

[4] Davis, M. M. (2003). Fundamentals of Operations Management. Boston: McGraw- Hill Irwin, Fourth Edition.

[5] De Landsheere, G. (1982). Introduction to research in education, Paris.

[6] Depelteau, F. (2000). The approach of a research in the human sciences. Brussels, Rue des Minimes

[7] Gorard, S. (2013). Research Design: Robust approaches for the social sciences, London, 218.

[8] Gorney. L. (1981). Queuing Theory: A Problem Solving Approach, New York.

[9] Karlin, S. (1969). A first Course in Stochastic Process. Academic Press, New York.

[10] Kleinrock, L. (1975). Queuing Systems, Volume I. Theory, Wiley, New York.

[11] Kleinrock, L. (1976). Queuing Systems, Volume II: Computer Applications. Wiley, New York.

[12] Petit Larousse, (1982). Paris.

[13] Sundarapandian, V. (2009). Queuing Theory. Probability, Statistics and Queuing Theory. PHI Learning, First edition. New Delhi.

[14] Taha, A. H. (2005). Operation Research: An Introduction. Delhi: Pearson Education, Inc.,Seven Edition. New Delhi.

[15] Allen, A.O. (1975). Elements of Queuing theory for system design, IBM Systems J. 14 (2).

[16] Babes, M. (1991). Out-patient Queues at the Ibn-Rochd Health Centre. Journal of Operational Research, 1086-1087.

[17] Belson, G. V. (1988). Waiting Times Scheduled Patient in the Presence of Emergency Request. Journal of Operational Management.

[18] Cochran, K.J. (2006). A Multi-stage Stochastic Methodology for Whole Hospital Bed Planning Under Peak Loading. International Journal of Industrial and Systems Engineering, 8-35.

[19] Davis, M.M. (1990). A framework for Relating Waiting Time and Customer Satisfaction in a Service Operation. Journal of Services Marketing, 61-69.

[20] Hall, W. R. (2006), Patient Flow: The New Queuing Theory for Healthcare. OR/Ms, Today. California.

[21] Hillier, F. S. and F. D. Lo. (1971). Tables for multi server queuing systems involving Erlang distributions, Tech, Rep. 31, December 28. Department of Operational Research, Stanford University, Stanford, California.

[22] Kandemir - Caues, C., Cauas, L. (2007). An Application of Queuing Theory to the Relationship between Insulin Level and Number of Insulin Receptors. Turkish Journal of Biochemistry, 32-38.

[23] Kingman, J. F. C. (1962). On queues in heavy traffic, J. Roy. Statist. Soc. Ser. B 24, 383-392.

[24] Köollerström, J. (1974). Heavy traffic theory for queues with several servers, I, J. Appl. Probability 11, 544-552.

[25] Nosek, A.R., Wislon, P. J. (2001). Queuing Theory and Customer Satisfaction: A Review of Terminology, Trends and Applications to Pharmacy Practice, Hospital Pharmacy, 275-279.

[26] Schlechter, (2009). Hershey Medical Center to open redesigned emergency room. The Patriot-News.

[27] Singh, V. (2011). Use of Queuing Models in Health Care, Department of Health Policy and Management, University of Arkanses for medical science. International Journal of Computing and Business Research. p 1-2